Problema (1)

1) $Dom[f(x)] = [-2, 2]$, dominio

$$y(x) = \begin{cases} a(x+2)^2 , & -2 \le x < 0 \quad \text{(parabola)} \\ \sqrt{-x^2-b} , & 0 \le x < 1 \quad \text{(circonferenza)} \\ \sqrt{x^2+c} , & 1 \le x \le 2 \quad \text{(iperbole)} \end{cases}$$

Voglio $y(x)$ continua in $x_0 = 0$ e $x_0 = 1$

$$\begin{cases} a(0+2)^2 = \sqrt{-b} = 1 \longrightarrow b = -1 , \; a = 1/4 \\ \sqrt{-1-b} = \sqrt{1+c} \longrightarrow c = -1 \end{cases}$$

da cui $$y(x) = \begin{cases} \frac{(x+2)^2}{4} , & -2 \le x < 0 \\ \sqrt{-x^2+1} , & 0 \le x < 1 \\ \sqrt{x^2-1} , & 1 \le x \le 2 \end{cases}$$

$$\frac{dy(x)}{dx} = \begin{cases} \dfrac{x+2}{2}, & -2 \le x < 0 \\ \dfrac{-x}{\sqrt{-x^2+1}}, & 0 \le x < 1 \\ \dfrac{x}{\sqrt{x^2-1}}, & 1 \le x \le 2 \end{cases}$$

$y'(x)$ non è continua in $x_0 = 0$ e $x_0 = 1$

quindi non è derivabile

$$\begin{cases} \dfrac{0+2}{2} = 1 \neq 0 , & \text{punto di discontinuità} \\ -\infty \neq 2 , & \text{punto di cuspide} \end{cases}$$

Scrivo le equazioni delle rette $t(x) = y(x_0) + y'(x_0)(x - x_0)$

in $x_0 = -2 \longrightarrow t(x) = 0$

$x_0 = 0 \longrightarrow \nexists\ t(x)$, derivata discontinua

$x_0 = 1 \longrightarrow \nexists\ t(x)$, derivata infinita

$x_0 = 2 \longrightarrow t(x) = 3 + 4(x-2)$

b)

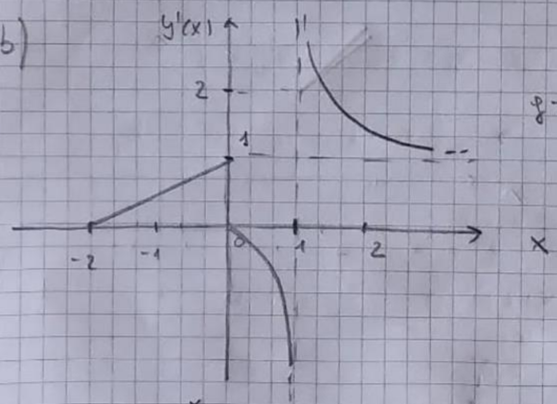


grafico derivata

$F(x) = \int_{-2}^{x} f(t)\, dt \longrightarrow F''(x) = f'(x)$

$$\begin{cases} f'(x) > 0 \text{ in } [-2, 0) \text{ e } (1, 2] \\ f'(x) < 0 \text{ in } (0, 1) \end{cases}$$

c) $y = \frac{(x+2)^2}{4}$ in $[-2,0]$

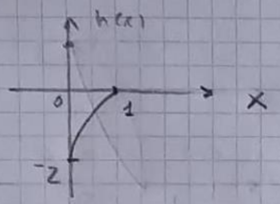
y è biunivoca in $[-2,0]$ quindi è invertibile

$4y = (x+2)^2 \longrightarrow x+2 = \pm 2\sqrt{y}$, $0<y<1$ in $[-2,0]$

$x = \pm 2(\sqrt{y}-1) < 0$

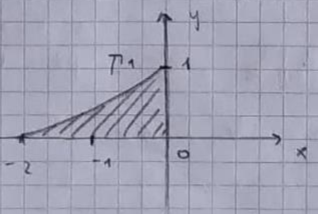
quindi prendo la soluzione $x = 2(\sqrt{y}-1)$

Chiamo $h(x) = +2(\sqrt{x}-1)$ la funzione inversa definita in $[0,1]$



$\frac{dh(x)}{dx} = +\frac{1}{\sqrt{x}}$ derivabile in $(0,1]$ infatti $h'(0) \xrightarrow{x \to 0} \infty$

d) S è la regione delimitata fra Γ_1 e gli assi cartesiani



$y(x) = \frac{(x+2)^2}{4}$

Voglio trovare $k \in [-2,0]$ tale che

$$\int_{-2}^{k} f(x)\,dx = \int_{k}^{0} f(x)\,dx$$

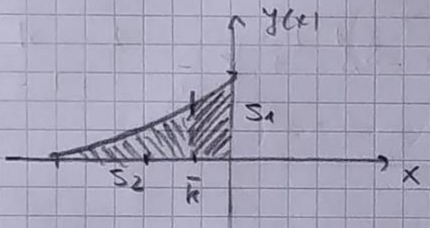
$$\int_{-2}^{k} \left(\frac{x^2}{4} + x + 1\right) dx = \int_{k}^{0} \left(\frac{x^2}{4} + x + 1\right) dx$$

$$\left(\frac{x^3}{12} + \frac{x^2}{2} + x\right)\Bigg|_{-2}^{k} = \left(\frac{x^3}{12} + \frac{x^2}{2} + x\right)\Bigg|_{k}^{0}$$

$$\frac{k^3}{12} + \frac{k^2}{2} + k + \frac{8^{\,2}}{\cancel{12}_{\,3}} - \cancel{2} + \cancel{2} = 0 - \frac{k^3}{12} - \frac{k^2}{2} - k$$

$$\frac{k^3}{6} + k^2 + 2k + \frac{2}{3} = 0 \quad \text{con} \quad k \in [-2, 0]$$

$$k^3 + 6k^2 + 12k + 4 = 0 \quad \text{in} \quad \bar{k} = 2^{2/3} - 2 \simeq -0,41$$



$$S = S_1 + S_2$$

Fonte: Studenti.it - https://www.studenti.it/problema-2-svolto-della-traccia-di-matematica-seconda-prova-2023.html